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Application of SCBZ Property In Manpower Planning Model

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Abstract
The exit of personnel which otherwise known as wastage leads to the depletion of manpower available with the organization. Recruitments could not be initiated immediately after every occasion of exit of personnel since it involves expenditure. If recruits are not made after exit of the personnel, the accumulating shortage of personnel on successive occasions when exceeds a particular level called the threshold level, results in the breakdown of the organization. This in turn makes the recruitments imperative. In this paper assuming that the threshold level is random variable in view of the dynamic nature of the market condition. It satisfies the Setting the Clock Back to Zero (SCBZ) property introduced by [3]. Using the shock model and Cumulative Damage Process (CDP) ofreliability theory the expected duration of the breakdown of the organization is obtained under the assumption that the threshold posses the SCBZ, the expected time to recruitment is obtained using shock model approach.
Keywords : Shock model, SCBZ, threshold,CDP.

Introduction

Exit of personnel which in other words known as wastage is an important aspect in the study of manpower planning. Many models have been discussed using different kinds of wastage and also different types of distributions depicting wastage. Such models could be seen in [1] and [2].

Cumulative Damage Process (CDP) is related to the shock models in reliability theory. The basic idea is that accumulating random amount of damages due to shocks in successive epochs leads to the breakdown of system when the total damage crosses a random threshold level. Assuming the exit of personnel as shocks to the manpower availability (in terms of man hours) in an organisational system the expected duration to the breakdown of the organisation due to the depletion of manpower is studied, taking a threshold level which in other words can be called as the breakdown point of the system. The breakdown point or the level can also be interpreted as that point at which the immediate recruitment is necessitated to makeup the manpower loss suffered cumulatively on successive occasions. Assuming exponential threshold [6] have obtained the expected duration to the breakdown/recruitment. In this paer instead of exponential distribution manifests Lack of Memory Property (LMP), the threshold distribution which is assumed to have Setting the Clock Back to Zero (SCBZ) property [4,5], and a comparison of this model with the model having exponential distribution is also given using numerical illustrations.

Assumptions of the Model

- Exit of persons from an organisation takes place whenever the policy decisions regarding targets, incentives and promotions are made.
- The exit of every person from the organisation results in a random amount of depletion of manpower (in man hours).
- The process of depletion is linear and cumulative.
- The inter arrival times between successive occasions of wastage are i.i.d. random variables.
- If the total depletion exceeds a threshold level Y which is itself a random variable, the breakdown of the organisation occurs. In other words recruitment becomes inevitable.
- The process which generates the exits the sequence of depletions and the threshold are mutually independent.

Notations

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X_i : a continuous random variable denoting the amount of damage/depletion caused to the system due to the exit of persons on the ith occasion of policy announcement, $i = 1, 2, \ldots k$ and X_i's are i. i.d. and $X_i = X$ for all i {Eg. $X_i \sim \exp(\alpha)$, for all i}.

Y : a continuous random variable denoting the threshold level having SCBZ property.

g(.) : the probability density function of X.

$g_k(.)$: the k-fold convolution of g(.) i.e., p.d.f. of $\sum_{i=1}^{k} X_i$

T : a continuous r.v denoting time to breakdown of the system.

τ_o : truncation point of the r.v. Y.

g*(.): Laplace transform of g(.).

g_k*(.): Laplace transform of g_k(.).

h(.) : the p.d.f. of random threshold level which has SCBZ property, and

H(.) : is the corresponding c.d.f.

U : a continuous random variable density the interarrival times between decision epochs decision epochs.

f(.) : p.d.f. of random variable corresponding c.d.f. F(.).

$F_k(t)$: the k-fold convolution function of F(.).

S(.) : the survivor function as P [T > t].

L(t) : 1 – S(t).

$V_k(t)$: probability that there are exactly 'k' policy decisions in (0, t].

Results

Let Y be the random variable which has the cdf defined as

$$H(y) = \begin{cases} 1 - e^{-\theta_1 y} & ; y \leq \tau_o \\ 1 - e^{-\theta_1 \tau_o} \; e^{-\theta_2 (y-\tau_o)} & ; \tau_o < y \end{cases} \qquad (1)$$

It can be verified that the random variable Y has a distribution which satisfies the SCBZ property, for some fixed τ_o by virtue of the fact that

$$\frac{S(x+\tau_o, \theta)}{S(\tau_o, \theta)} = S(x, \theta^*)$$

where S(x, θ) denotes the survivor function corresponding the random variable Y. Assuming the truncation level τ_o itself a random variable which follows exponential distribution with parameter λ, it can be shown that

$$\therefore H(y) = 1 - \frac{\theta_1 - \theta_2}{\lambda + \theta_1 - \theta_2} e^{-(\theta_1 + \lambda) y} - \frac{\lambda}{\lambda + \theta_1 - \theta_2} e^{-\theta_2 y} \qquad (2)$$

and

$$h(y) = \left[\frac{(\theta_1 - \theta_2)(\lambda + \theta_1)}{\lambda + \theta_1 - \theta_2} e^{-(\theta_1 + \lambda) y} + \frac{\theta_2 \lambda}{\lambda + \theta_1 - \theta_2} e^{-\theta_2 y} \right]$$

$$\therefore h(y) = p\,(\theta_1 + \lambda)\, e^{-(\theta_1 + \lambda) y} + q \theta_2 e^{-\theta_2 y} \qquad (3)$$

where $p = \frac{\theta_1 - \theta_2}{\lambda + \theta_1 - \theta_2}$, $q = \frac{\lambda}{\lambda + \theta_1 - \theta_2}$ and p + q = 1.

Now,

$P(X_1 + X_2 + \ldots + X_k < Y) =$ Probability [the system does not fail, after k epochs of exits].

$$= \int_0^\infty g_k(x) \; [1 - H(x)] \; dx$$

$$= \int_{o}^{\infty} g_k(x)\left[pe^{-(\theta_1+\lambda)x} + qe^{-\theta_2 x}\right] dx$$

$$= p\, g_k^*(\theta_1 + \lambda) + q\, g_k^*(\theta_2)$$

Hence $P\left(\sum_{i=1}^{k} Xi < Y\right) = p\,[g^*(\theta_1 + \lambda)]^k + q\,[g^*(\theta_2)]^k$ (4)

since $g_k^*(.) = [g^*(.)]^k$ by the fact that X_i's are i.i.d.

The survivor function

P(T > t) = Probability that the system survives beyond 't'

$$= \sum_{k=0}^{\infty} \text{P (there are exactly k instants of exits in (0, t]}$$

$$* \text{ P (the system does not fail in (0, t]}$$

$$= \sum_{k=0}^{\infty} V_k(t) P\left(\sum_{i=1}^{k} Xi < Y\right) \quad (5)$$

Taking Laplace transform of L(t), we get

$$L^*(s) = p\,[1 - g^*(\theta_1+\lambda)] \sum_{k=1}^{\infty} [g^*(\theta_1 + \lambda)]^{k-1} [f^*(s)]^k$$

$$+ q\,[1 - g^*(\theta_2)] \sum_{k=1}^{\infty} [g^*(\theta_2)]^{k-1} [f^*(s)]^k \quad (6)$$

where $[f^*(s)]^k$ is Laplace transform of $F_k(t)$ since the inter arrival times are i.i.d. The above equation can be rewritten as,

$$L^*(s) = p\,[1 - g^*(\theta_1+\lambda)]\, f^*(s) \sum_{k=1}^{\infty} [g^*(\theta_1 + \lambda)\, f^*(s)]^{k-1}$$

$$+ q\,[1 - g^*(\theta_2)]\, f^*(s) \sum_{k=0}^{\infty} [g^*(\theta_2)\, f^*(s)]^{k-1}$$

$$= \frac{p\left[1 - g^*(\theta_1 + \lambda)\right] f^*(s)}{1 - g^*(\theta_1 + \lambda) f^*(s)} + \frac{q\left[1 - g^*(\theta_2)\right] f^*(s)}{1 - g^*(\theta_2) f^*(s)} \text{ on simplification} \quad (7)$$

$$E(T) = -\frac{d}{ds} L^*(s)\bigg|_{s=0} = \mu_1' \quad (8)$$

$$E(T^2) = \frac{d^2}{ds^2} L^*(s)\bigg|_{s=0} = \mu_2', \quad (9)$$

from which V(T) can be obtained.

Case (i)

It may be observed that if it is assumed that the threshold Y follows simply the exponential distribution with parameter θ (without SCBZ property) than it can be shown that

$$L^*(s) = \frac{\left[1 - g^*(\theta)\right] \; f^*(s)}{1 - g^*(\theta) f^*(s)} \qquad (10)$$

from which E(T) and V(T) can be computed, using (8) and (9).

Let the random variable 'U' denoting interarrival was follow exponential with parameter c and let the random variable Y

Now, $f^*(s) = \dfrac{c}{c+s}$ $g^*(\theta) = \dfrac{\alpha}{\alpha+\theta}$ follow exponential with parameter of

Substituting this in (4.10) we get

$$L^*(s) = \frac{\left[1 - \frac{\alpha}{\alpha+\theta}\right] \frac{c}{c+s}}{1 - \frac{\alpha}{\alpha+\theta} \cdot \frac{c}{c+s}} \qquad (11)$$

$$= \frac{c\theta}{s(\alpha+\theta)+c\theta}$$

$$\frac{d}{ds} L^*(s) = -\frac{c\theta.(\alpha+\theta)}{[s(\alpha+\theta+c\theta)]^2}$$

Using equation (11) and (12) we get,

$$E(T) = \frac{(\alpha \;+\; \theta)}{c\theta} \qquad (12)$$

$$\frac{d^2}{ds^2} L^*(s) = -\frac{2c\theta.(\alpha+\theta)^2}{[s(\alpha+\theta+\rho\theta)]^3} = E(T_2)$$

Using equation (4.9) we get

$$V(T) = \frac{(\alpha \;+\; \theta)^2}{c^2\theta^2} \qquad (13)$$

To obtain V(T) using (12) and (13)

$$\text{Therefore } V(T) = \frac{(\alpha \;+\; \theta)^2}{c^2\theta^2} \qquad (14)$$

on simplification.

Case (ii)

Here Y satisfies the SCBZ property with parameters θ_1 and θ_2. Then in this case, assuming U~exp(c) , X~exp(α). Hence,

$$L^*(s) = \frac{p\left[1 - g^*(\theta_1+\lambda)\right]\frac{c}{c+s}}{1 - g^*(\theta_1+\lambda)\frac{c}{c+s}} + \frac{q\left[1 - g^*(\theta_2)\right]\frac{c}{c+s}}{1 - g^*(\theta_2)\frac{c}{c+s}} \qquad (15)$$

$$= p\,\frac{(\lambda+\theta_1)c}{c(\lambda+\theta_1)+s(\alpha+\lambda+\theta_1)} + q.\frac{\theta_2.c}{c\theta_2+s(\alpha+\theta_2)}$$

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$$\frac{d}{ds} L^*(s) = -p. \frac{(\lambda+\theta_1)c \ \ (\alpha+\lambda+\theta_1)}{[(\lambda+\theta_1)+s(\alpha+\lambda+\theta_1)]^2} - q \ q.\frac{\theta_2.c \ \ (\alpha+\theta_2)}{[c\theta_2+s(\alpha+\theta_2)]^2}$$

Using equation (8) we get

Then in this case, assuming U~exp(c), X~exp(α), we get,

$$E(T) = p \frac{\alpha+\lambda+\theta_1}{c[\lambda+\theta_1]} + q\frac{\alpha+\theta_2}{c\theta_2} \qquad (16)$$

$$\frac{d^2}{ds} L^*(s) = \frac{2c(\lambda+\theta_1)(\alpha+\lambda+\theta_1)^2}{[(\lambda+\theta_1)+s(\alpha+\lambda+\theta_1)]^3}$$

Using equation (9) we get

$$E(T^2) = p\left(\frac{\alpha+\lambda+\theta_1}{c[\lambda+\theta_1]}\right)^2 + q\left(\frac{\alpha+\theta_2}{c\theta_2}\right)^2 \qquad (17)$$

Now, using equation (16) and (17) to we get,

$$V(T)= \frac{1}{c^2}\left[p\left(\frac{\alpha+\lambda+\theta_1}{[\lambda+\theta_1]}\right)^2(2-p) + q\left(\frac{\alpha+\lambda+\theta_2}{[\theta_2]}\right)^2(2-q) - 2pq\left(\frac{\alpha+\lambda+\theta_1}{[\lambda+\theta_1]}\right)\left(\frac{\alpha+\lambda+\theta_2}{[\theta_2]}\right)\right]$$

on simplification. (18)

Conclusion

From the Table 1 and the corresponding figures (Figures1.1,1.2,1.3) we could observe the difference in the values of the expectation of T when the threshold distribution as SCBZ property and when it does not poses the same. In the case of the threshold with SCBZ property with $\theta_1 = 0.4$ and $\theta_2 = 0.2$ with $\gamma = 0.6$, $\alpha = 1.0$, as C increases E(T) decreases in the case of the threshold with nm – SCBZ property with $\alpha = 1.0$, $\theta = 0.4$, as C increases E(T) also decreases. In both the cases the behavior is found to be the same but the expected values in the SCBZ property always lie above the corresponding expected value of the non-SCBZ property. The same kind of phenomenan is observed even then the values of the parameters are changed. The behavior of variance of T for the variation in C in the ase of the threshold distribution with SCBZ property and non-SCBZ can be observed in Table 2 and the corresponding figures (Figures2.1,2.2,2.3). The variance of the non-SCBZ case are initially higher is then the corresponding values of the SCBZ case. However, as C increases the variances declining simultaneously and converse together. The same kind of phenomenon could be observed irrespective of the changes in the values of the other parameters. Further, it may be concluded that when the threshold distribution has SCBZ property with parameter hange the expected values are becoming larges compare to the threshold with non-SCBZ property.

Table 1

	Expectation of T

	SCBZ	Non SCBZ	SCBZ	Non SCBZ	SCBZ	Non SCBZ
	λ=0.6, θ_1=0.4, θ_2=0.2 α=1.0	θ=0.4, α=1.0	λ=0.6, θ_1=0.4, θ_2=0.2 α=5.0	θ=0.4, α=5.0	λ=0.6, θ_1=0.4, θ_2=0.2 α=9.0	θ=0.4, α=9.0
1	5.0000	3.5000	21.00	13.50	37.00	23.50
2	2.5000	1.7500	10.50	6.75	18.50	11.75
3	1.6667	1.6667	7.00	4.50	12.33	7.83
4	1.2500	0.8750	5.25	3.37	9.25	5.88
5	1.0000	0.7000	4.20	2.70	7.40	4.70
6	0.8333	0.5833	3.50	2.25	6.17	3.92
7	0.7143	0.5000	3.00	1.93	5.28	3.36
8	0.6250	0.4375	2.63	1.69	4.63	2.94
9	0.5556	0.3889	2.33	1.50	4.11	2.61
10	0.5000	0.3500	2.10	1.35	3.70	2.35

Table 2

C	Variance of T					
	SCBZ	Non SCBZ	SCBZ	Non SCBZ	SCBZ	Non SCBZ
	λ=0.6, θ_1=0.4, θ_2=0.2 α=1.0	θ=0.4, α=1.0	λ=0.6, θ_1=0.4, θ_2=0.2 α=5.0	θ=0.4, α=5.0	λ=0.6, θ_1=0.4, θ_2=0.2 α=9.0	θ=0.4, α=9.0
1	6.0001	12.2500	150.00	182.25	486.00	552.25
2	1.5000	3.0630	37.50	45.56	121.50	138.06
3	0.6667	1.3611	16.67	20.25	54.00	61.36
4	0.3750	0.7658	9.38	11.39	30.38	34.52
5	0.2400	0.4900	6.00	7.29	19.44	22.09
6	0.1670	0.3403	4.17	5.06	13.50	15.34
7	1.2245	0.2500	3.06	3.72	9.92	11.27
8	0.0937	0.1914	2.34	2.85	7.59	8.63
9	0.0741	0.1512	1.85	2.25	6.00	6.82
10	0.0600	0.1225	1.50	1.82	4.86	5.52

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Figure 1.1

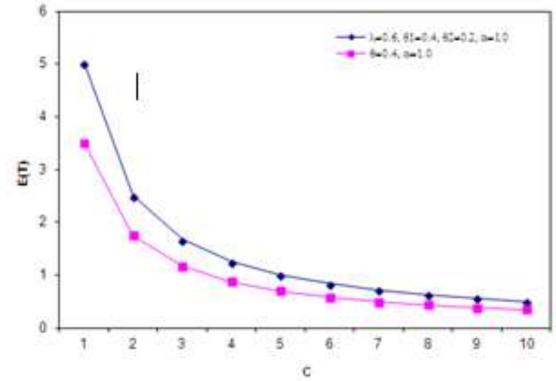


Figure 1.2

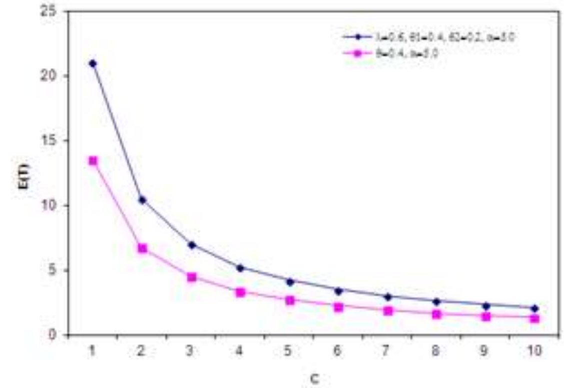


Figure 1.3

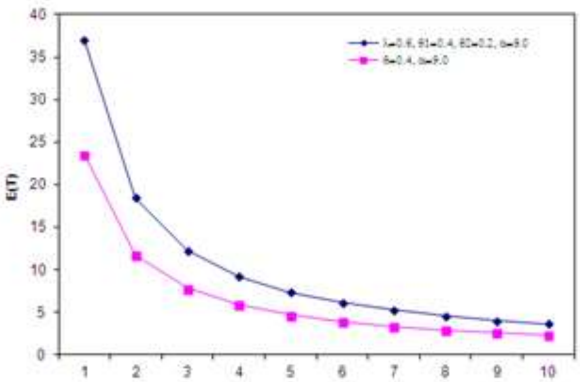


Figure 2.1

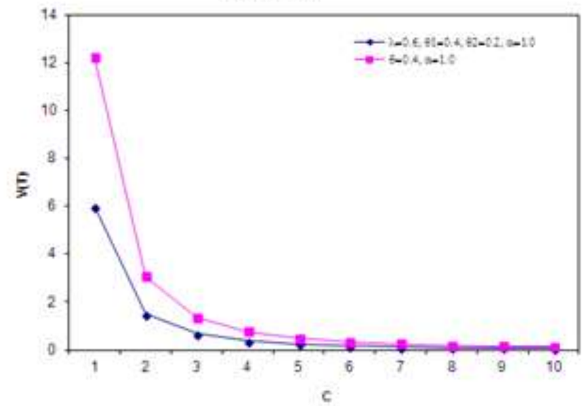


Figure 2.2

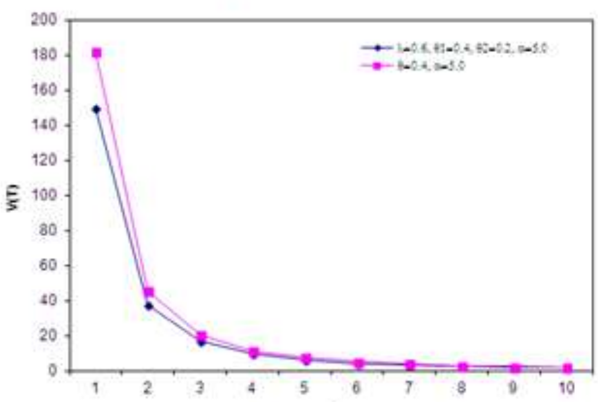
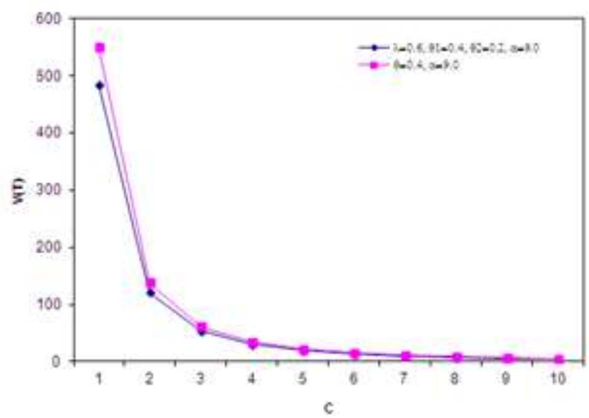


Figure 2.3



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